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MULTIPLE – EQUATION MODELS OF ORDERED DEPENDENT VARIABLES IN EXPLORATION OF THE RESULTS OF REHABILITATION OF LOCOMOTIVE ORGAN DISORDERS

Jerzy Grosman[1] , Mieczysław Kowerski[2]

ABSTRACT

In the present paper concerning the analysis of the factors determining patient's self-service during the admission and release from hospital a two-equation model of ordered dependent variables is proposed. These types of models are especially useful when the results of rehabilitation of locomotive organ disorders are not described by means of exact values obtained by mechanical measurements but they are described by means of qualitative valuation (ranking) made by a therapist when the distances between neighbouring ranks are not known. The advantages of the proposed model were presented on the basis of the results of estimation based on data of 4063 patients of hospitals from Mazowieckie and Warmińsko-Mazurskie provinces.

Key words: Rehabilitation of locomotive organ disorders, Weiss test, multiple-equation model of ordered dependent variable, maximum likelihood estimation, McFadden determination coefficient - pseudo R^2, count R-squared, probability of the norm in the self-service test.

1. Preface

The results of rehabilitation of locomotive organ disorders are often described not by exact values of measurements with specific devices but by values, ranked from the lowest to the highest, for which the distances between consecutive ranks are not known (e.g. in the Weiss test the patients are classified using the following ranks: 1 – patient without pain; 2 – periodic pain, low severity, not reported by the patient; 3 – postexercise pain after a long walk, etc.). In this situation classic regression method should not be used to model the relationships between the

[1] Akademia Wychowania Fizycznego w Warszawie.
[2] Wyższa Szkoła Zarządzania i Administracji w Zamościu, ul. Akademicka 4, 22-400 Zamość, tel. 84 6776711, mowerski@wszia.edu.pl.

results of rehabilitation and their determinants. Models of ordered dependent variable seem to be much more relevant.

In the present paper the models of ordered dependent variable have been used to describe the self-service status of a patient. Since the self-service status is measured both at the hospital admission and at the hospital release of a patient, a two-equation recursive model is proposed, in which the first equation describes the patient status at the admission and the second equation – at the release from the hospital.

The source of information for all values of dependent and explanatory variables in this paper are the Weiss test results for 4063 patients.

2. Weiss test

The Weiss test is used to assess the locomotive ability and the social and occupational situation of a patient [Weiss, Zembaty 1983, Grossman 1987, Górski et al. 1985, Grossman 1988]. It is used not only for patients with stroke but also for geriatric patients.

As a result of numerous clinical trials nine test were selected as representative for the whole research:

- pain test (variable P_1 at the admission and variable K_1 at the release),
- motion test (variable P_2 at the admission and variable K_2 at the release),
- muscular strength test (variable P_3 at the admission and variable K_3 at the release),
- fitness test (variable P_4 at the admission and variable K_4 at the release),
- locomotion test (variable P_5 at the admission and variable K_5 at the release),
- wheelchair locomotion test (variable P_6 at the admission and variable K_6 at the release),
- self-service test (variable P_7 at the admission and variable K_7 at the release),
- hand-grip test (variable P_8 at the admission and variable K_8 at the release),
- occupational adaptation test (variable P_9 at the admission and variable K_9 at the release)

Each of these test has a six-rank assessment scale, where 1 is the norm. Higher values mean an unfavourable (negative) intensification of the variable.

3. Data

The data used in the present paper has been collected from hospitals and health centres in Warsaw and the provinces of Mazowieckie and Warminsko-Mazurskie during last 20 years. The information about patients (age, sex, length of hospitalization in months) and the Weiss test results at the admission and at the release was recorded for 4063 persons[1].

Table 1. Basic statistics of the population

Variable	N	Mean	Std. Dev.	Median	Mode	Min	Max.	Q1	Q3	Skewness	Kurtosis
Sex	4063	1.44	0.5914	1	1	1	22	1	2	10.51	357.95
Age	4063	43.97	17.9941	45	45	10	92	27	59	0.06	-1.09
Length of hospitalization	4063	47.41	39.0778	39	35	1	589	22	60	2.86	17.90
P_1	4063	2.65	1.4984	3	1	1	6	1	4	0.49	-0.77
P_2	4063	3.37	1.5667	3	2	1	6	2	5	0.13	-1.06
P_3	4063	3.08	1.4697	3	3	1	6	2	4	0.40	-0.61
P_4	4063	2.25	1.3641	2	1	1	6	1	3	1.10	0.52
P_5	4063	3.57	1.8683	3	6	1	6	2	6	0.05	-1.43
P_6	4063	2.57	1.7985	2	1	1	6	1	4	0.77	-0.86
P_7	4063	3.15	1.9434	3	1	1	6	1	5	0.27	-1.48
P_8	4063	1.59	1.2580	1	1	1	6	1	1	2.27	4.24
P_9	4063	4.24	2.1084	6	6	1	6	2	6	-0.60	-1.39
K_1	4063	1.55	0.8174	1	1	1	6	1	2	1.56	2.41
K_2	4063	2.59	1.4309	2	2	1	6	1	4	0.75	-0.31
K_3	4063	2.42	1.3541	2	1	1	6	1	3	0.90	0.19
K_4	4063	1.67	0.9481	1	1	1	6	1	2	1.74	3.47
K_5	4063	2.58	1.5056	2	1	1	6	1	3	0.87	-0.09
K_6	4063	1.65	1.1276	1	1	1	6	1	2	1.87	2.95
K_7	4063	2.19	1.4574	2	1	1	6	1	3	1.06	0.06
K_8	4063	1.45	1.0559	1	1	1	6	1	1	2.67	6.71
K_9	4063	3.42	2.0999	3	1	1	6	1	6	0.08	-1.69

Note: Variables: $P_1, P_2,\ldots,P_9$ represent the Weiss test results at the admission to the hospital while the $K_1, K_2,\ldots,K_9$ variables represent the Weiss test results at the release from the hospital.

Source: calculated with Statistica 6.0

[1] Patients who died during the rehabilitation process were excluded from the panel.

It was assumed in this paper that the dependent variables are: the self-service test results at the admission (P_7) and at the release (K_7) from the hospital.

Table 2. Correlation coefficients between the dependent variables and potential explanatory variables

Variable	P_7	K_7
Sex	-0.0633*	-0.0441*
Age	0.3573*	0.2046*
Length of hospitalization	x	0.1359*
P_1	0.1926*	0.0992*
P_2	0.5070*	0.4633*
P_3	0.4572*	0.4709*
P_4	0.5231*	0.4683*
P_5	0.7073*	0.5173*
P_6	0.6732*	0.4783*
P_7	1.0000	0.6509*
P_8	0.3931*	0.3717*
P_9	0.6448*	0.4578*
K_1	x	0.0815*
K_2	x	0.4937*
K_3	x	0.5221*
K_4	x	0.5140*
K_5	x	0.6739*
K_6	x	0.6025*
K_7	x	1.0000
K_8	x	0.3892*
K_9	x	0.5956*

Note: (*) means that the correlation coefficient is significant at the level of at least .05.
Source: calculated with Statistica 6.0.

The values of Pearson's linear correlation coefficients[1] for P_7 show that there is a statistically significant relationship between the dependent variable and all potential explanatory variables. The older the patient is, the poorer (higher value

[1] The qualitative (rank) nature of the test results makes the Pearson's linear correlation coefficient not the best tool to assess dependencies between the variables. However, it was chosen to present the results in the preliminary analysis because it is very popular. The more so because many research showed that the assessment of relationships based on the Pearson's correlation coefficients gives approximate results to assessments based on qualitative measures of associations.

of test) self-service results at the admission are achieved. At the same time, the poorer the results of other tests are, the poorer self-service results at the admission are recorded. The K_7 variable also shows a statistically significant positive relationship with age and the test results both at the admission and at the release. To summarise: the older the person is and the higher test results at the admission are, the higher self-service test score is at the release of a patient. The statistically significant positive value of the correlation coefficient between the self-service test at the release and the length of stay in the hospital may be surprising. This can be explained as follows: the patients who stayed longer in the hospital were released in a worse self-service condition. However, this conclusion is based just on single coefficient of correlation, which does not reflect the relation between the length of hospitalization and the patient's age or self-service status at the admission. This issue will be subject to further analysis.

The relationships between qualitative discreet variables (ranks) should be analysed with the Pearson χ^2 test [Stanisz 2001, s. 286-287].

Table 3. Results of the Pearson χ^2 test for the self-service test and the other tests

Variable	P_7		K_7	
	χ^2	p	χ^2	p
P_1	327.616	<0.000001	142.840	<0.000001
P_2	1283.420	<0.000001	1190.780	<0.000001
P_3	1117.670	<0.000001	1142.210	<0.000001
P_4	1456.920	<0.000001	1244.480	<0.000001
P_5	2782.570	<0.000001	1423.680	<0.000001
P_6	2462.490	<0.000001	1146.290	<0.000001
P_7	x	x	3132.180	<0.000001
P_8	930.575	<0.000001	897.134	<0.000001
P_9	2379.180	<0.000001	1227.830	<0.000001
K_1	x	x	106.511	<0.000001
K_2	x	x	1275.980	<0.000001
K_3	x	x	1358.270	<0.000001
K_4	x	x	1689.340	<0.000001
K_5	x	x	2638.060	<0.000001
K_6	x	x	2304.370	<0.000001
K_8	x	x	1009.920	<0.000001
K_9	x	x	2094.300	<0.000001

Source: calculated with Statistica 6.0.

The results of the χ^2 – Pearson test for a relationship of a self-service test at the admission and the other Weiss tests show that these relationships are very close and statistically significant at .000001. Similar situation is between the self-service tests at the admission and the other Weiss tests – these relationships are statistically significant at least at .000001. The self-service tests at the release can also be an example of such dependence.

Table 4. Self-service tests at the admission vs. at the release from a hospital

Value	At the admission		At the release		Change of (%)
	No. of patients	(%)	No. of patients	(%)	
1	1308	32.2	1935	47.6	15.4
2	551	13.6	748	18.4	4.8
3	537	13.2	602	14.8	1.6
4	341	8.4	363	8.9	0.5
5	524	12.9	264	6.5	-6.4
6	802	19.7	151	3.7	-16.0
Total	4063	100.0	4063	100.0	0.0

Source: authors' calculation.

The treatment applied caused an improvement of patient self-service. At the admission to a hospital only one third of patients showed a normal status of self-service, whereas at the release this number grew to almost a half. On the other hand, the number of patients with rank 6 (who require a permanent care) dropped from 19.7% at the admission to 3.7% at the release (16 points).

4. Two-equation model of patient's self-service

Taking into account the presented relationships between dependent and potential explanatory variables it is proposed to use a two-equation recursive model where the self-service statuses at the admission and at the release play a role of dependent variables.

These variables may adopt one out of six possible states (values) of patient's self-service:

1. Full ability to perform all daily indoor and outdoor activities
2. Ability to perform basic daily indoor activities without special equipment
3. Partial ability to perform daily activities, which does not require assistance from other persons
4. Independence in daily activities, but special equipment is needed
5. Partial independence in daily activities, which requires other person's assistance and special equipment in some activities

6. Fully dependent, permanent care by other person required

The recursive model is composed of two equations.

First equation describes patient's self-service status at the admission to a hospital

$$P_7 = f\left(\text{Sex}, \text{Age}, P_1, P_2, P_3, P_4, P_5, P_6, P_8, P_9, \varepsilon_1\right) \quad (1)$$

The second equation describes patient's self-service at the release (K_7) and considers the length of stay in the hospital and the self-service status at the admission, which is represented – typically for recursive models – by the theoretical values of the first dependent variable, estimated from the first equation [Syczewska 2003, pp. 209-210] $\hat{P}_7$. In this way the estimated value of P_7 is representative for the test results at the admission and the patient's age.

$$K_7 = f\left(\text{Sex}, \text{Age}, \text{Length of hosp}, \hat{P}_7, K_1, K_2, K_3, K_4, K_5, K_6, K_8, K_9, \varepsilon_2\right) \quad (2)$$

Each dependent value adopts six values ranked from the lowest to the highest and the distances between consecutive ranks are unknown. In this situation the most appropriate approach to explain the variation of the dependent variable (reasons for different values of the self-service test) is the use of ordered dependent variable models [Greene 2003, pp. 736-740]. In case of both tests (descriptions of daily activities) the assumption of equal distances between neighbouring categories, e.g. between rank 2 and rank 3 and between rank 3 and rank 4, can hardly be maintained. It is assumed, however, that there is a model of self-service – latent variable[1] y^*:

$$y_i^* = \mathbf{X}_i^T \beta + \varepsilon_i \quad (3)$$

where:

y_i^* – value of latent dependent variable for an *i*-th patient,

$\mathbf{X}_i^T$ – a vector of values of independent variables for an *i*-th patient,

β – a vector of structural parameters of the model,

ε_i – an error term,

on the basis of which the assessment of patient's self-service is made both at the admission and at the release.

The value of the ordered dependent value y_i depends then on the value of the latent variable y_i^*, according to the rule[2]

[1] *Y* means any dependent variable, P_7 and K_7 in our case. The **X** vector is a vector of independent variables, which in our case includes the results of other tests and other characteristic of the patients (sex, age, length of stay in a hospital).

[2] EViews 5.1 User's Guide Quantitative Micro Software, LLC, Irvine CA, s. 639.

$$y_i = \begin{cases} 1 \text{ dla } y_i^* \le \gamma_1 \\ 2 \text{ dla } \gamma_1 < y_i^* \le \gamma_2 \\ 3 \text{ dla } \gamma_2 < y_i^* \le \gamma_3 \\ \dots\dots\dots\dots\dots\dots\dots\dots. \\ M \text{ dla } \gamma_M < y_i^* \end{cases} \tag{4}$$

It does not matter that the ranks were chosen arbitrarily and may adopt different values in other models. The only requirement for ordered variables is to meet the following rule:

$$\text{If } y_i^* < y_j^* \text{ then } y_i < y_j \tag{5}$$

A big advantage of this model, especially in medical research, is that the results may be interpreted as probabilities. The probability of each rank of the Y variable is as follows:

$$\begin{gathered} \Pr\left(y_i = 0 \middle| x_i, \beta, \gamma\right) = F\left(\gamma_1 - x_i^T \beta\right) \\ \Pr\left(y_i = 1 \middle| x_i, \beta, \gamma\right) = F\left(\gamma_2 - x_i^T \beta\right) - F\left(\gamma_1 - x_i^T \beta\right) \\ \Pr\left(y_i = 2 \middle| x_i, \beta, \gamma\right) = F\left(\gamma_3 - x_i^T \beta\right) - F\left(\gamma_2 - x_i^T \beta\right) \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ \Pr\left(y_i = M \middle| x_i, \beta, \gamma\right) = 1 - F\left(\gamma_M - x_i^T \beta\right) \end{gathered} \tag{6}$$

where F is a distribution function of the random variable ε.

The γ values are the boundary points and are estimated together with the β parameters by means of the maximum likelihood method through maximizing the log likelihood.

The maximization of the likelihood function may be done using a few methods. The EViews software offers Newton-Raphson and Goldfeld-Quandt (quadratic hill climbing) methods. In most cases it is assumed that the models of ordered dependent variable are logit or probit models.

If we assume that the ordered dependent variable adopts a probit model, then the F function is a distribution function of a standardized normal distribution, which value may be read directly from the standardized normal distribution tables. If we assume that the ordered dependent variable adopts a logit model, then the F function is a logistic distribution function, which values may be calculated using the following formula:

$$F(y) = \frac{e^y}{1 + e^y} \tag{7}$$

While choosing the most appropriate model the criterion of maximisation of McFadden determination coefficient was applied [McFadden 2002, p. 65], together with the rule that the model should consist of only such variables, which parameters are significant and coincident [Hellwig 1976].

5. Results of the estimation of the two-equation patient's self-service model

In the present paper logit models of the ordered dependent variable were adopted.

5.1. Estimation of the first equation[1]

When the criterion of significance and coincidence of all parameters was applied, the optimum model describing the P_7 variable includes Age, Sex, and the results of 7 tests at the admission.

1 An example of an effective application of the ordered dependent variable models in economic research is the work of Bielak J., M. Kowerski, An approach to identify the factors determining the assessment of the regional labour market in the Lubelskie province, Prace i Materiały Instytutu Rozwoju Gospodarczego nr 80, SGH, Warsaw 2008, pp. 233–258 (in polish).

Table 5. The results of estimation of the first equation of the patient's self-service model at the admission

Dependent Variable: P_7

Method: ML – Ordered Logit (Quadratic hill climbing)

Sample: 4063

Included observations: 4063

Number of ordered indicator values: 6

Estimation settings: tol= 0.0001

Convergence achieved after 6 iterations

Covariance matrix computed using second derivatives

Estimations of structural parameters:

	Coefficient	Std. Error	z-Statistic	Prob,
Sex	0.1191	0.0604	1.9738	0.048400
Age	0.0231	0.0021	11.1381	<0.000001
P_2	0.2155	0.0289	7.4508	<0.000001
P_3	0.1407	0.0313	4.4893	<0.000001
P_4	0.1406	0.0301	4.6687	<0.000001
P_5	0.5073	0.0259	19.5766	<0.000001
P_6	0.4138	0.0258	16.0272	<0.000001
P_8	0.5137	0.0340	15.1179	<0.000001
P_9	0.3209	0.0213	15.0702	<0.000001
Limit Points				
LIMIT_2:C(1)	5.8970	0.1737	33.9478	<0.000001
LIMIT_3:C(2)	7.1503	0.1859	38.4608	<0.000001
LIMIT_4:C(3)	8.3850	0.1995	42.0382	<0.000001
LIMIT_5:C(4)	9.2419	0.2092	44.1779	<0.000001
LIMIT_6:C(5)	10.7596	0.2271	47.3848	<0.000001
Goodness-of-fit				
Akaike info criterion	2.2881	Schwarz criterion		2.3098
Log likelihood	-4634.227	Hannan-Quinn criter.		2.2958
Restr. log likelihood	-6889.575	Avg. log likelihood		-1.1406
LR statistic (9 df)	4510.696	LR index (Pseudo-R^2)		0.3274
Probability(LR stat)	0.000000			

Source: calculations with the eViews software.

The estimated model shows that all variables are stimulants, i.e. the higher value they adopt, the higher theoretical value of self-service test is. In other words, since the lowest rank means a norm, the worse the self-service test results are. In terms of likelihood it means that the higher values of independent variables, the higher probability of increased value of the self-service test at the admission. The McFadden coefficient is .3274, which is not a high value, but the significance of the likelihood ratio, measuring the significance of the whole set of variables in the model, proves high explanatory potential of the model. On the other hand, numerous research confirmed that the comparatively low value McFadden determination coefficients (for example exceeding .25) are usually sufficient to accept the goodness-of-fit of the model [Gruszczyński 2002, p. 55]. Additionally, the count R-squared, a percentage of correct indications of the model out of all indications [Maddala 1992, p. 334] is 65.1%.

The model allows analysis of relationship between age and the self-service status at the admission. In simulations it was assumed that a hypothetical patient has the following values of the independent variables:

1. 1st quartiles of variables for the whole population
2. Medians of variables for the whole population
3. Means of variables for the whole population
4. 3rd quartiles of variables for the whole population.

Table 6. Values of independent variables adopted for simulation

Variable	Q_1	Median	Mean	Q_3
P_2	2	3	3.37	5
P_3	2	3	3.08	4
P_4	1	2	2.25	3
P_5	2	3	3.57	6
P_6	1	2	2.57	4
P_8	1	1	1.59	1
P_9	2	6	4.24	6

Source: authors' calculation.

Table 7. Change of self-service probabilities depending on other tests, age and sex

Age	Men						Women					
	P_{71}	P_{72}	P_{73}	P_{74}	P_{75}	P_{76}	P_{71}	P_{72}	P_{73}	P_{74}	P_{75}	P_{76}
Independent variables (tests) at the level of the 1st quartile												
20	**0.868**	0.091	0.029	0.007	0.004	0.001	**0.853**	0.100	0.033	0.008	0.005	0.001
35	**0.823**	0.119	0.040	0.010	0.006	0.002	**0.805**	0.131	0.045	0.011	0.007	0.002
50	**0.766**	0.154	0.055	0.014	0.008	0.002	**0.744**	0.166	0.062	0.016	0.009	0.003
65	**0.699**	0.192	0.075	0.020	0.012	0.003	**0.673**	0.205	0.083	0.022	0.013	0.004
80	**0.622**	0.230	0.100	0.027	0.016	0.005	**0.593**	0.243	0.110	0.030	0.018	0.005
Independent variables (tests) at the level of the median												
20	**0.305**	0.301	0.235	0.085	0.057	0.017	0.281	**0.297**	0.247	0.093	0.063	0.019
35	0.237	**0.284**	0.268	0.109	0.078	0.024	0.216	0.275	**0.277**	0.118	0.086	0.027
50	0.180	0.255	**0.291**	0.136	0.104	0.034	0.163	0.243	**0.295**	0.145	0.115	0.038
65	0.135	0.218	**0.299***	0.163	0.137	0.047	0.121	0.205	**0.298***	0.172	0.150	0.053
80	0.099	0.179	**0.292**	0.187	0.177	0.066	0.089	0.166	**0.286**	0.194	0.192	0.073
Independent variables (tests) at the level of the mean												
20	0.230	**0.281**	0.271	0.112	0.081	0.025	0.209	0.272	**0.280**	0.121	0.089	0.028
35	0.174	0.251	**0.293**	0.139	0.108	0.035	0.158	0.238	**0.297**	0.149	0.119	0.040
50	0.130	0.213	**0.299***	0.167	0.142	0.049	0.117	0.200	**0.298***	0.175	0.155	0.055
65	0.095	0.174	**0.290**	0.190	0.182	0.068	0.086	0.161	**0.283**	0.197	0.197	0.076
80	0.070	0.138	**0.266**	0.206	0.227	0.094	0.062	0.126	**0.255**	0.209	0.243	0.104
Independent variables (tests) at the level of the 3rd quartile												
20	0.020	0.047	0.131	0.170	**0.358**	0.273	0.018	0.042	0.120	0.161	**0.361***	0.297
35	0.014	0.034	0.101	0.143	**0.361***	0.347	0.013	0.031	0.091	0.134	0.358	**0.374**
50	0.010	0.025	0.076	0.116	0.345	**0.429**	0.009	0.022	0.068	0.107	0.336	**0.458**
65	0.007	0.018	0.056	0.091	0.314	**0.515**	0.006	0.016	0.050	0.083	0.301	**0.544**
80	0.005	0.013	0.041	0.069	0.273	**0.600**	0.005	0.011	0.036	0.063	0.257	**0.628**

Note: 1 P_{71} means a probability of adopting a value of 1 in the self-service test, P_{72} means a probability of adopting a value of 2 in the self-service test, etc. 2 (*) inflection point. 3 The maximum probabilities for a given age are bolded, this means that the variable P_7 adopts a values from a column with bolded probability.

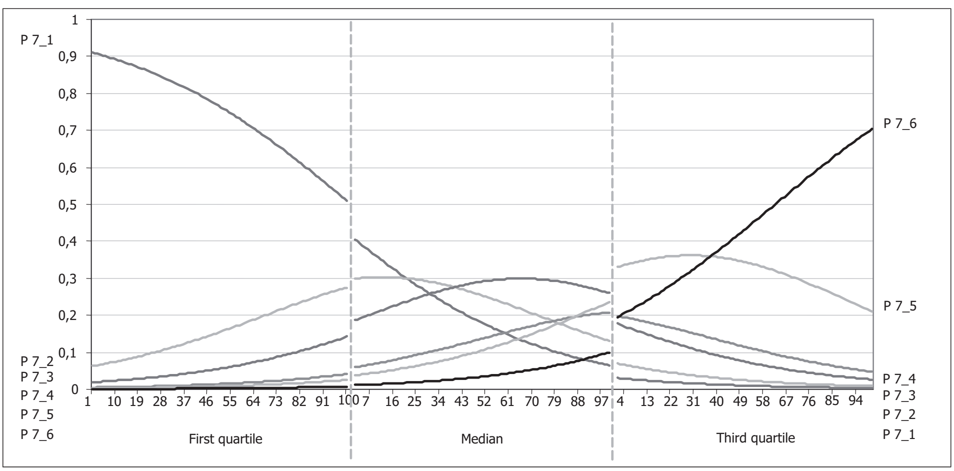
Source: authors' calculation

The simulation research shows that regardless of the results of other tests included in the model, the probability of good self-service declines and the probability of poorer self-service grows with ageing process. When the other tests reach values of their 1st quartiles, which means the norm for P_4, P_6 and P_8, or the value of 2 (at least good) for P_2, P_3, P_5 and P_9, the probability of normal self-service is .868 for a 20-year old man and .853 for a 20-year old woman. The probability of norm for a 60-year old man is .699, and .673 for a 60-year old woman. In case of 80-year old persons these probabilities are .622 for a man and .593 for a woman. When the results of other tests worsen, the probability of norm is lower and the probability of poorer self-service increases. For example, in case of 35-year old man, for whom the other tests reached their median value, the probability of norm is .237 and is lower than the probability of that the self-service test adopts the value of 2 (probability of .284) [1].

In case of a 80-year old woman, for whom the other tests adopted values of the 3rd quartile (poor or very poor) the probability of norm is only .005, which means that only 5 women out of 1000 would have a self-service test result equal to the norm. For 63 it would be 4, for 257 the value would be 5, but the majority of women (628) would obtain the worst value of the self-service test.

It is also worth mentioning that for all groups the probability of norm is lower for women than for men at the same age.

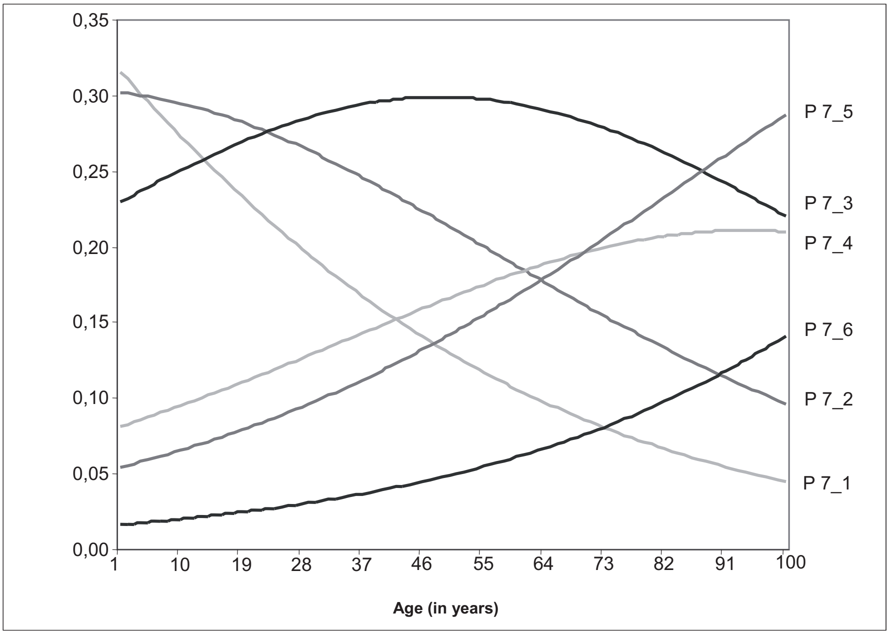
Figure 1. Probabilities for a given self-service status at the admission (P_7) at the assumed test results for P_2, P_3, P_4, P_5, P_6, P_8, P_9 at the admission versus age. Men.



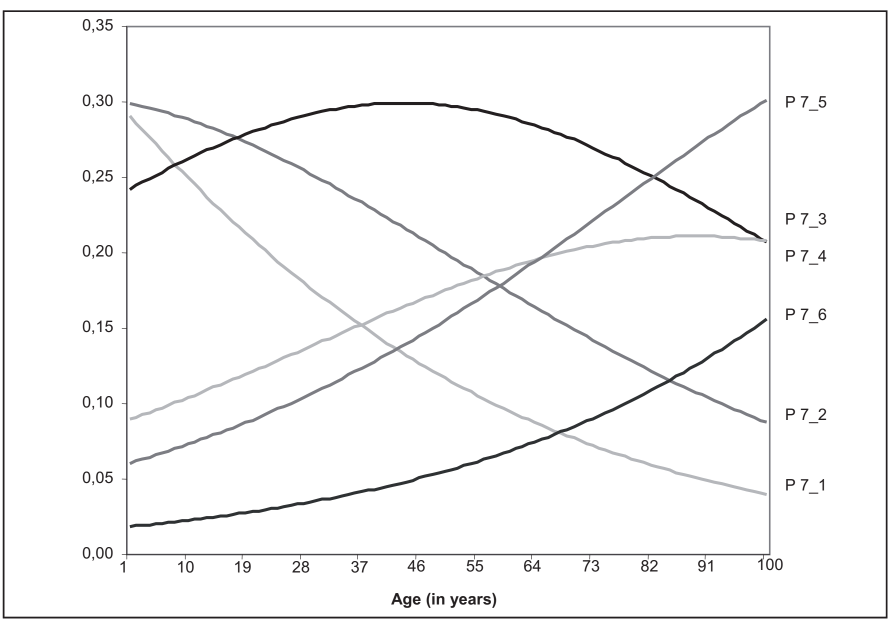
1 If we change again from a theoretical probability to a theoretical rank, the latter for the variable P_7 will be 2 because the probability for 2 is the highest.

Figure 2. Probabilities for a given self-service status at the admission (P_7) assuming that the test results for P_2, P_3, P_4, P_5, P_6, P_8, P_9 at the admission reach the mean value, versus age.

Men



Women



The detailed analysis of changes in probability of particular self-service ranks versus age (incl. Fig. 1 and Fig. 2) leads to following conclusions:

1. The probability of the norm (1) decreases with age along a „reversed" logistic curve
2. The probability of the worst self-service rank (6) increases with age along a logistic curve
3. The probabilities of the other self-service ranks (from 2 to 5) increase up to a certain maximum and then decrease[1]; the "earliest" maximum is reached by the curve representing the probability of that the self-service rank is 2, then by the curve representing the probability of that the self-service rank is 3 and finally by the curve representing the probability of that the self-service rank is 5.

5.2. Estimation results for the second equation

According to the accepted methodology of building a recursive model the second equation describes changes of the self-service assessments at the release (K_7) but the patient's status at the admission is described by theoretical values of $\hat{P}_7$. There are two possible ways to calculate the theoretical value of the dependent variable in the first equation:

- the $\hat{P}_7$ variable adopts the rank, for which the probability estimated by the model is the highest
- the $\hat{P}_7$ variable is a mean rank weighted by the probabilities of the ranks

$$\hat{P}_7 = 1 \circ P(P_7 = 1) + 2 \circ P(P_2 = 2) + 3 \circ P(P_7 = 3) + 4 \circ P(P_7 = 4) + 5 \circ P(P_5 = 5) + 6 \circ P(P_7 = 6)$$

The second option was used in this paper.

[1] The analysis of probability functions for three-rank answers was presented in Kowerski M., On the Informative Value of „Remain Unchanged", Barometr Regionalny nr 4 (14), Wyższa Szkoła Zarządzania i Administracji w Zamościu, Zamość 2008, pp. 47–62 (in polish).

Table 8. Estimation results for the patient's self-service at the release

Dependent Variable: K_7

Method: ML - Ordered Logit (Quadratic hill climbing)

Sample: 1 4063

Included observations: 4063

Number of ordered indicator values: 6

Estimation settings: tol= 0.00010

Convergence achieved after 8 iterations

Covariance matrix computed using second derivatives

Structural parameters

	Coefficient	Std. Error	z-Statistic	Prob.
Sex	0.2046	0.0668	3.0652	0.002175
Age	0.0063	0.0023	2.6907	0.007131
P_{7teor}	0.2408	0.0328	7.3456	<0.000001
K_2	0.1910	0.0326	5.8588	<0.000001
K_3	0.2339	0.0352	6.6487	<0.000001
K_4	0.2510	0.0409	6.1349	<0.000001
K_5	0.5055	0.0330	15.3153	<0.000001
K_6	0.3833	0.0368	10.4107	<0.000001
K_8	0.3064	0.0326	9.3887	<0.000001
K_9	0.3474	0.0225	15.4546	<0.000001
Limit Points				
LIMIT_2:C(1)	5.9213	0.1789	33.1022	<0.000001
LIMIT_3:C(2)	7.5226	0.1956	38.4567	<0.000001
LIMIT_4:C(3)	9.1167	0.2143	42.5391	<0.000001
LIMIT_5:C(4)	10.4956	0.2318	45.2824	<0.000001
LIMIT_6:C(5)	12.4096	0.2654	46.7615	<0.000001
Goodness-of-fit				
Akaike info criterion	1.9866	Schwarz criterion		2.0099
Log likelihood	-4020.713	Hannan-Quinn criter.		1.9948
Restr. log likelihood	-5946.303	Avg. log likelihood		-0.9896
LR statistic (10 df)	3851.178	LR index (Pseudo-R^2)		0.3238
Probability(LR stat)	0.000000			

Source: calculated with eViews software.

In the model estimated here all parameters are significant and their signs are as expected. The probability that the self-service test is closer to the norm at the release is higher, if the patient is younger, the self-service test result at the admission was better and the results of other tests at the release were better. The McFadden determination coefficient is at the level of that for the first equation, but the accuracy of estimation for the second equation, measured by the count R-squared, is much higher and reaches 80.0%. A simulation of self-service status at the release was run depending on the theoretical values of the self-service test at the admission, age and sex, assuming that the results of other tests at the release are at the median level.

Table 9. Medians for the tests at the release

Variable	Median
K_2	2
K_3	2
K_4	1
K_5	2
K_6	1
K_8	1
K_9	3

Table 10. Changes of the self-service probabilities at the release depending on the theoretical values of the self-service test at the admission, age and sex, assuming that the results of other tests at the release are at the median level

Age	Men						Women					
	K_{71}	K_{72}	K_{73}	K_{74}	K_{75}	K_{76}	K_{71}	K_{72}	K_{73}	K_{74}	K_{75}	K_{76}
The theoretical value of the self-service test at the admission is 1												
20	**0.819**	0.139	0.034	0.007	0.002	0.000	**0.786**	0.162	0.041	0.008	0.002	0.000
35	**0.804**	0.149	0.037	0.007	0.002	0.000	**0.770**	0.173	0.045	0.009	0.003	0.000
50	**0.789**	0.160	0.040	0.008	0.002	0.000	**0.753**	0.185	0.049	0.010	0.003	0.000
65	**0.773**	0.171	0.044	0.009	0.003	0.000	**0.735**	0.197	0.053	0.011	0.003	0.001
80	**0.756**	0.183	0.048	0.010	0.003	0.000	**0.716**	0.210	0.058	0.012	0.003	0.001
The theoretical value of the self-service test at the admission is 2												
20	**0.780**	0.166	0.042	0.009	0.002	0.000	**0.743**	0.192	0.051	0.010	0.003	0.001
35	**0.763**	0.178	0.046	0.009	0.003	0.000	**0.725**	0.204	0.056	0.011	0.003	0.001
50	**0.746**	0.190	0.050	0.010	0.003	0.001	**0.705**	0.217	0.061	0.013	0.004	0.001
65	**0.728**	0.202	0.055	0.011	0.003	0.001	**0.686**	0.230	0.066	0.014	0.004	0.001
80	**0.709**	0.215	0.060	0.012	0.004	0.001	**0.665**	0.243	0.072	0.015	0.004	0.001
The theoretical value of the self-service test at the admission is 3												
20	**0.736**	0.197	0.053	0.011	0.003	0.001	**0.694**	0.224	0.064	0.013	0.004	0.001
35	**0.717**	0.209	0.058	0.012	0.003	0.001	**0.674**	0.237	0.069	0.014	0.004	0.001
50	**0.698**	0.222	0.063	0.013	0.004	0.001	**0.653**	0.250	0.075	0.016	0.005	0.001
65	**0.678**	0.235	0.068	0.014	0.004	0.001	**0.631**	0.263	0.082	0.017	0.005	0.001
80	**0.657**	0.248	0.074	0.016	0.005	0.001	**0.609**	0.276	0.089	0.019	0.006	0.001
The theoretical value of the self-service test at the admission is 4												
20	**0.687**	0.229	0.066	0.014	0.004	0.001	**0.641**	0.258	0.079	0.017	0.005	0.001
35	**0.666**	0.242	0.072	0.015	0.004	0.001	**0.619**	0.271	0.086	0.018	0.005	0.001
50	**0.645**	0.255	0.078	0.016	0.005	0.001	**0.597**	0.283	0.093	0.020	0.006	0.001
65	**0.623**	0.268	0.085	0.018	0.005	0.001	**0.574**	0.296	0.101	0.022	0.006	0.001
80	**0.604**	0.279	0.091	0.019	0.006	0.001	**0.551**	0.308	0.109	0.024	0.007	0.001
The theoretical value of the self-service test at the admission is 5												
20	**0.633**	0.263	0.082	0.017	0.005	0.001	**0.584**	0.290	0.097	0.021	0.006	0.001
35	**0.611**	0.275	0.089	0.019	0.006	0.001	**0.561**	0.303	0.105	0.023	0.007	0.001
50	**0.588**	0.288	0.096	0.021	0.006	0.001	**0.538**	0.315	0.114	0.025	0.007	0.001
65	**0.565**	0.301	0.104	0.023	0.007	0.001	**0.514**	0.326	0.123	0.028	0.008	0.001
80	**0.542**	0.313	0.112	0.025	0.007	0.001	**0.491**	0.336	0.132	0.030	0.009	0.002
The theoretical value of the self-service test at the admission is 6												
20	**0.575**	0.295	0.100	0.022	0.006	0.001	**0.525**	0.321	0.119	0.027	0.008	0.001
35	**0.552**	0.307	0.108	0.024	0.007	0.001	**0.501**	0.332	0.128	0.029	0.009	0.002
50	**0.529**	0.319	0.117	0.026	0.008	0.001	**0.478**	0.342	0.138	0.032	0.009	0.002
65	**0.505**	0.330	0.126	0.029	0.009	0.001	**0.454**	0.351	0.148	0.035	0.010	0.002
80	**0.482**	0.340	0.136	0.031	0.009	0.002	**0.431**	0.359	0.159	0.038	0.011	0.002

Source: authors' calculation.

Probability of the norm at the release of a 20-year old man, whose self-service test rank at the admission was at the level of the norm and the other tests at the release reached the median, was .819. If a 20-year man's self-service status at the admission was 2 and the other tests at the release were equal to their medians, the probability of the norm at the release is .780. In case of a 20-year old woman the results are a bit lower and reach .786 and .743, respectively. The probability of the norm at the release decreases with patient's age and the self-service level at the admission. For example, the probability of the norm for a 80-year old man, whose self-service status at the admission was 6 and the other tests at the release reached the median, was .482. For a 80-year old woman it is even lower and reaches .431.

Such high probabilities result from good results of the other tests at the release (median levels).

Table 11. Test means at the release

Variable	Mean
K_2	2.59
K_3	2.42
K_4	1.67
K_5	2.58
K_6	1.65
K_8	1.45
K_9	3.42

Source: authors' calculation.

Table 12. Changes of the self-service probabilities at the release depending on the theoretical values of the self-service test at the admission, age and sex, assuming that the results of other tests at the release are at the mean level

Age	Men						Women					
	K_{71}	K_{72}	K_{73}	K_{74}	K_{75}	K_{76}	K_{71}	K_{72}	K_{73}	K_{74}	K_{75}	K_{76}
The theoretical value of the self-service test at the admission is 1												
20	**0.574**	0.296	0.101	0.022	0.006	0.001	**0.523**	0.321	0.119	0.027	0.008	0.001
35	**0.551**	0.308	0.109	0.024	0.007	0.001	**0.500**	0.332	0.129	0.029	0.009	0.002
50	**0.527**	0.320	0.118	0.026	0.008	0.001	**0.476**	0.342	0.138	0.032	0.010	0.002
65	**0.504**	0.330	0.127	0.029	0.009	0.001	**0.453**	0.351	0.149	0.035	0.010	0.002
80	**0.481**	0.341	0.137	0.031	0.009	0.002	**0.430**	0.359	0.160	0.038	0.011	0.002
The theoretical value of the self-service test at the admission is 2												
20	**0.514**	0.326	0.123	0.028	0.008	0.001	**0.463**	0.347	0.144	0.033	0.010	0.002
35	**0.491**	0.336	0.132	0.030	0.009	0.002	**0.440**	0.356	0.155	0.037	0.011	0.002
50	**0.467**	0.346	0.142	0.033	0.010	0.002	**0.417**	0.363	0.166	0.040	0.012	0.002
65	**0.444**	0.354	0.153	0.036	0.011	0.002	**0.394**	0.369	0.177	0.044	0.013	0.002
80	**0.421**	0.362	0.164	0.039	0.012	0.002	0.372	**0.374**	0.189	0.048	0.015	0.003
The theoretical value of the self-service test at the admission is 3												
20	**0.454**	0.351	0.148	0.035	0.010	0.002	**0.404**	0.367	0.172	0.042	0.013	0.002
35	**0.431**	0.359	0.159	0.038	0.011	0.002	**0.382**	0.372	0.184	0.046	0.014	0.002
50	**0.408**	0.366	0.170	0.041	0.013	0.002	0.360	**0.376**	0.196	0.050	0.015	0.003
65	**0.393**	0.369	0.178	0.044	0.013	0.002	0.338	**0.379**	0.209	0.054	0.017	0.003
80	0.364	**0.376**	0.194	0.049	0.015	0.003	0.318	**0.380**	0.221	0.059	0.018	0.003
The theoretical value of the self-service test at the admission is 4												
20	**0.395**	0.369	0.177	0.043	0.013	0.002	0.348	**0.378**	0.203	0.052	0.016	0.003
35	0.373	**0.374**	0.189	0.047	0.014	0.003	0.327	**0.380**	0.216	0.057	0.018	0.003
50	0.352	**0.377**	0.201	0.052	0.016	0.003	0.306	**0.380***	0.229	0.062	0.019	0.003
65	0.330	**0.379**	0.213	0.056	0.017	0.003	0.287	**0.379**	0.242	0.067	0.021	0.004
80	0.310	**0.380**	0.226	0.061	0.019	0.003	0.268	**0.377**	0.255	0.073	0.023	0.004
The theoretical value of the self-service test at the admission is 5												
20	0.340	**0.379**	0.208	0.054	0.017	0.003	0.295	**0.380**	0.236	0.065	0.020	0.004
35	0.319	**0.380***	0.221	0.059	0.018	0.003	0.276	**0.378**	0.249	0.071	0.022	0.004
50	0.299	**0.380**	0.234	0.064	0.020	0.004	0.258	**0.375**	0.262	0.077	0.024	0.004
65	0.279	**0.378**	0.247	0.070	0.022	0.004	0.240	**0.370**	0.275	0.083	0.027	0.005
80	0.261	**0.376**	0.260	0.076	0.024	0.004	0.223	**0.365**	0.287	0.090	0.029	0.005
The theoretical value of the self-service test at the admission is 6												
20	0.288	**0.379**	0.241	0.067	0.021	0.004	0.248	**0.373**	0.269	0.080	0.026	0.005
35	0.269	**0.377**	0.254	0.073	0.023	0.004	0.231	**0.367**	0.282	0.087	0.028	0.005
50	0.251	**0.373**	0.267	0.079	0.025	0.005	0.214	**0.361**	0.294	0.094	0.031	0.006
65	0.234	**0.368**	0.280	0.086	0.028	0.005	0.199	**0.353**	0.307	0.102	0.034	0.006
80	0.217	**0.362**	0.292	0.093	0.030	0.005	0.184	**0.344**	0.318	0.110	0.037	0.007

Source: authors' calculation.

When the results of other tests at the release worsen, the probabilities of the norm decrease. For example, the probability of the norm for a 80-year old man, whose self-service status at the admission was 6 and the other tests at the release reached the mean, was .362

Table 13. Changes of the probability of K_7 reaching a particular value when theoretical P_7 changes from 1 to 6 (based on data in the Table 12)

Age	Men						Women					
	K_{71}	K_{72}	K_{73}	K_{74}	K_{75}	K_{76}	K_{71}	K_{72}	K_{73}	K_{74}	K_{75}	K_{76}
20	-0.286	0.083	0.140	0.045	0.015	0.003	-0.276	0.051	0.150	0.054	0.018	0.003
35	-0.282	0.069	0.145	0.049	0.016	0.003	-0.269	0.035	0.153	0.058	0.020	0.004
50	-0.277	0.054	0.149	0.053	0.018	0.003	-0.262	0.019	0.156	0.062	0.021	0.004
65	-0.270	0.038	0.153	0.057	0.019	0.003	-0.254	0.002	0.158	0.067	0.023	0.004
80	**-0.263**	0.021	0.156	0.061	0.021	0.004	-0.245	-0.015	0.159	0.072	0.025	0.005

Source: authors' calculation.

Obviously, the worse self-service status at the admission, the lower probability of the norm at the release (assuming that the other test results are the same). For example, for 20-year old men, if the self-service status at the admission worsens from the norm (1) to total disability (6), the probability of the norm at the release decreases by .263, assuming that the other tests remain at the mean level.

6. Conclusions

The two-equation model estimated suggests that the models of ordered dependent variable are suitable to determine the factors affecting locomotive abilities of the patients. This model allows simulating the probabilities of the patient's self-service status both at the admission and at the release from a hospital depending on various factors describing a patient.

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